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THE STEADY-STATE WAVE PROCESS IN A PIEZOELECTRIC LAYER AND HALF-LAYER WEAKENED BY TUNNEL CUTS (ANTIPLANE DEFORMATION)†

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The antiplane dynamic problem of electro-elasticity for a piezoelectric layer and half-layer containing curvilinear tunnel cuts along their bases is investigated. Integral representations of the solutions are constructed, by means of which the corresponding boundary value problems are reduced to a singular integro-differential equation in the jump in the amplitude of the displacement on the cuts. The asymptotic form of the combined mechanical and electric fields in the neighbourhood of singular points is investigated. The results of a numerical realization of the algorithm which enables the effect of the excitation frequency, the curvature of the cut (the crack), the type of boundary conditions and the effect of connectedness of the fields on the stress intensity factor K_{III} are presented.

1. CONSIDER a piezoelectric layer $0 \leqslant x_1 \leqslant a$, $-\infty < x_2 < \infty$, $-\infty < x_3 < \infty$, weakened by tunnel cuts L_j $(j = 1, 2, \ldots, k)$ along the x_3 axis, referred to the crystallographic axes x_1, x_2, x_3. We will conventionally assume that the piezoelectric material is a transversely isotropic material with an axis of symmetry parallel to the x_3 axis (the crystal belongs to the $6mm$ hexagonal system, polarized in advance along the x_3 axis of the piezoelectric material).

We will assume that a monochromatic shear wave $u_3^{(0)} = \mathrm{Re}[U_3^{(0)}(x_1, x_2)\exp(-i\omega t)]$ is radiated from infinity and it is possible for a shear load $X_{3n}^{\pm} = \mathrm{Re}[X_3^{\pm}\exp(-i\omega t)]$, which varies harmonically with time and is constant along the x_3 axis, to act on the edges of the cuts while the bases of the layer are free from forces and are bounded by vacuum. We will assume that the curvatures of the contours L_j and the amplitudes $X_3^+ = -X_3^- = X_3$ are functions of the class H [1] on $L = \cup L_j$ and moreover $\cap L_l = \varnothing$ (Fig. 1).

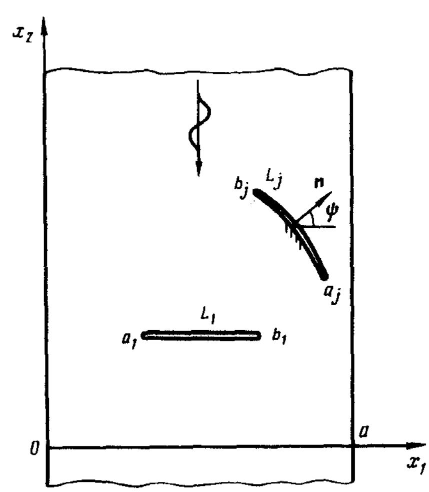

FIG. 1.

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In this formulation, in the layer with cuts there will be combined mechanical and electromechanical fields corresponding to antiplane deformation.

The complete system of equations has the form

$$\tau_{\nu 3}=c_{44}^E\partial_\nu u_3-e_{15}E_\nu,\quad D_\nu=e_{15}\partial_\nu u_3+\varepsilon_{11}^S E_\nu \qquad (1.1)$$
$$\partial_\nu=\partial/\partial x_\nu\quad (\nu=1,\,2)$$

$$\partial_1\tau_{13}+\partial_2\tau_{23}=\rho\frac{\partial^2 u_3}{\partial t^2} \qquad (1.2)$$

$$\partial_1E_2-\partial_2E_1+\mu\frac{\partial H_3}{\partial t}=0,\qquad \partial_1D_1+\partial_2D_2=0 \qquad (1.3)$$
$$\partial_2H_3=\frac{\partial D_1}{\partial t},\quad \partial_1H_3=-\frac{\partial D_2}{\partial t}$$

Here τ_{13}, τ_{23} and u_3 are the shear stresses and the displacement along the x_3 axis, E_1, E_2, H_3 and D_1, D_2 are the components of the electric and magnetic fields, respectively, and also of the electric induction vector, c_{44}^E is the shear modulus, e_{15} is the piezoelectric constant, ε_{11}^S and μ are the permittivity and magnetic permeability of the medium, and ρ is the density of the material. The electric boundary conditions on the edges of the cuts are taken in the form [3]

$$E_s^+=E_s^-,\quad D_n^+=D_n^- \qquad (1.4)$$

Here E_S and D_n are the tangential component of the electric field vector and the normal component of the electric induction vector, respectively.

The boundary conditions on the bases of the layer can be written in the form

$$\tau_{13}=0,\ D_1=0\ (x_1=0;\ a). \qquad (1.5)$$

Introducing the function Φ as given by

$$E_1=-\frac{e_{15}}{\varepsilon_{11}^S}\partial_1u_3+\partial_2\Phi,\qquad E_2=-\frac{e_{15}}{\varepsilon_{11}^S}\partial_2u_3-\partial_1\Phi,\qquad H_3=\varepsilon_{11}^S\frac{\partial\Phi}{\partial t} \qquad (1.6)$$

we arrive at the equations

$$\nabla^2u_3-\frac{1}{c^2}\frac{\partial^2u_3}{\partial t^2}=0,\qquad \nabla^2\Phi-\frac{1}{c_\alpha^2}\frac{\partial^2\Phi}{\partial t^2}=0 \qquad (1.7)$$
$$c^2=\frac{c_{44}^E(1+\varkappa_0^2)}{\rho},\qquad c_\alpha^2=\frac{1}{\mu\varepsilon_{11}^S},\qquad \varkappa_0^2=\frac{e_{15}^2}{c_{44}^E\varepsilon_{11}^S}$$

In the quasi-static approximation (for not very large cuts) over a wide range of angular frequencies ω we can assume that $\nabla^2\Phi=0$. By virtue of (1.1) and (1.6) we have

$$\tau_{13}=c_{44}^E(1+\varkappa_0^2)\partial_1u_3-e_{15}\partial_2\Phi,\quad D_1=\varepsilon_{11}^S\partial_2\Phi$$
$$\tau_{23}=c_{44}^E(1+\varkappa_0^2)\partial_2u_3+e_{15}\partial_1\Phi,\quad D_2=-\varepsilon_{11}^S\partial_1\Phi\qquad u_3=u_3^{(0)}+u_3^*$$

Here the quantity u_3^* represents the displacement field perturbed by the cuts. Assuming

$$u_3=\operatorname{Re}\,[U_3(x_1,\,x_2)e^{-i\omega t}],\quad \Phi=\operatorname{Re}\,[F(x_1,\,x_2)e^{-i\omega t}]$$
$$U_3=U_3^{(0)}+U_3^*,\quad U_3^{(0)}=\tau e^{-i\gamma_2x_2},\quad \gamma_2=\omega/c$$

we can represent the boundary conditions on the edges L_j in the form

$$c_{44}^E(1+\varkappa_0^2)\left\{e^{i\psi}\left(\frac{\partial U_3}{\partial\zeta}\right)^{\pm}+e^{-i\psi}\left(\frac{\partial U_3}{\partial\bar\zeta}\right)^{\pm}\right\}-$$
$$-ie_{15}\left\{e^{i\psi}\left(\frac{\partial F}{\partial\zeta}\right)^{\pm}-e^{-i\psi}\left(\frac{\partial F}{\partial\bar\zeta}\right)^{\pm}\right\}=\pm X_3^{\pm} \qquad (1.8)$$

$$\frac{ie_{15}}{\varepsilon_{11}{}^S}\left\{e^{i\psi}\left[\frac{\partial U_3}{\partial \zeta}\right]-e^{-i\psi}\left[\frac{\partial U_3}{\partial \bar\zeta}\right]\right\}+e^{i\psi}\left[\frac{\partial F}{\partial \zeta}\right]+$$
$$+e^{-i\psi}\left[\frac{\partial F}{\partial \bar\zeta}\right]=0,\quad e^{i\psi}\left[\frac{\partial F}{\partial \zeta}\right]-e^{-i\psi}\left[\frac{\partial F}{\partial \bar\zeta}\right]=0$$

$$\zeta=\xi_1+i\xi_2,\ \bar\zeta=\xi_1-i\xi_2,\ \zeta\in L_j,\ [g]=g^+-g^-\ (j=1,2,\ldots,k)$$

The upper sign relates to the left edge of L_j (for motion from its beginning a_j to the end b_j) and ψ is the angle between the positive normal to the left edge and the ox_1 axis.

2. The boundary-value problems (1.7) and (1.5) can be written in terms of the amplitudes

$$\nabla^2U_3+\gamma_2{}^2U_3=0;\ \partial_1U_3=0\ (x_1=0;\ a) \tag{2.1}$$

$$\nabla^2F=0;\ \partial_2F=0\ (x_1=0;\ a) \tag{2.2}$$

We will expand Green's function corresponding to problems (2.1), (2.2) in the form

$$\begin{aligned}G(x_1-\xi_1,x_2-\xi_2)&=\sum_{k=0}^{\infty}b_k(x_2-\xi_2)\cos\alpha_k\xi_1\cos\alpha_kx_1\\E(x_1-\xi_1,x_2-\xi_2)&=\sum_{k=1}^{\infty}d_k(x_2-\xi_2)\sin\alpha_k\xi_1\sin\alpha_kx_1\end{aligned} \tag{2.3}$$

$$\nabla^2G+\gamma_2{}^2G=\delta(x_1-\xi_1,\ x_2-\xi_2),\ \alpha_k=\pi k/a$$

$$\nabla^2E=\delta(x_1-\xi_1,\ x_2-\xi_2),\ \delta(x,\ y)=\delta(x)\delta(y)$$

where $\delta(x)$ is a $2a$-periodic Dirac delta function. Using the expansions

$$\delta(x_1-\xi_1)=\frac{1}{a}+\frac{2}{a}\sum_{k=1}^{\infty}\cos\alpha_k\xi_1\cos\alpha_kx_1$$

$$\delta(x_1-\xi_1)=\frac{2}{a}\sum_{k=1}^{\infty}\sin\alpha_k\xi_1\sin\alpha_kx_1$$

separating the variables in (2.1) and (2.2) and then using the procedure for determining the fundamental solution [4], we obtain

$$b_k=-\frac{1}{a\lambda_k}e^{-\lambda_k|x_2-\xi_2|},\quad b_0=\frac{1}{2ia\gamma_2}e^{i\gamma_2|x_2-\xi_2|} \tag{2.4}$$

$$d_k=-\frac{1}{a\alpha_k}e^{-\alpha_k|x_2-\xi_2|},\quad \lambda_k=\begin{cases}\sqrt{\alpha_k{}^2-\gamma_2{}^2}, & \gamma_2<\alpha_k\\-i\sqrt{\gamma_2{}^2-\alpha_k{}^2}, & \gamma_2>\alpha_k\end{cases}$$

$$(k=1,\ 2,\ \ldots)$$

The series for the function E in (2.3), by making use of the relation

$$\sum_{m=1}^{\infty}\frac{e^{-m|x|}}{m}\cos my=\frac{|x|}{2}-\frac{1}{2}\ln[2(\operatorname{ch}x-\cos y)] \tag{2.5}$$

is easily summed to give

$$E(x_1-\xi_1,x_2-\xi_2)=\frac{1}{2\pi}\ln\left|\frac{\sin{}^1/_2\pi(\zeta-z)/a}{\sin{}^1/_2\pi(\zeta+\bar z)/a}\right| \tag{2.6}$$

$$z=x_1+ix_2,\ \bar z=x_1-ix_2$$

To separate the principal part of the function G we will write Green's function G_0 of the leading operator in the Holmholtz equation (2.3). Summing the corresponding series using (2.5) we obtain

$$G_0 = -\frac{1}{a}\sum_{m=1}^{\infty} a_m(x_1,\xi_1)\,e^{-\alpha_m|x_2-\xi_2|} = -\frac{|x_2-\xi_2|}{2a} + $$
$$+\frac{1}{2\pi}\ln\left|4\sin\frac{\pi(\zeta-z)}{2a}\sin\frac{\pi(\zeta+\bar z)}{2a}\right|,\quad a_m = \frac{\cos\alpha_m\xi_1\cos\alpha_m x_1}{\alpha_m} \tag{2.7}$$

Taking (2.4) and (2.7) into account, we can represent (2.3) in the following final form:

$$G(x_1-\xi_1, x_2-\xi_2) = G_0 + G_1,\qquad G_1 = \frac{1}{2ia\gamma_2}e^{i\gamma_2|x_2-\xi_2|} -$$
$$-\frac{1}{a}\sum_{m=1}^{\infty} c_m(x_2-\xi_2)\cos\alpha_m\xi_1\cos\alpha_m x_1 \tag{2.8}$$
$$c_m(x_2-\xi_2) = \frac{1}{\lambda_m}e^{-\lambda_m|x_2-\xi_2|} - \frac{1}{\alpha_m}e^{-\alpha_m|x_2-\xi_2|}$$
$$(m=1,2,\ldots)$$

Hence, the functions E and G defined in (2.6) and (2.8) are Green's functions of the boundary-value problems (2.1) and (2.2) for a strip. The radiation condition for problem (2.1) and the attenuation condition for problem (2.2) are satisfied. After separating the principal part in (2.3), the general term of the series in (2.8) decays at the point $z=\zeta$ as m^{-3}.

3. Using the well-known reflection method [5], Green's function constructed above for a layer can be generalized to the case of a half-layer ($0\leqslant x_1\leqslant a$, $0\leqslant x_2<\infty$, $-\infty<x_3<\infty$). We will assume that the side bases of the half-layer are free from forces and are bounded by vacuum, while on the boundary $x_2=0$ the following types of mechanical and electric conditions are possible: no forces, contact with vacuum

$$\tau_{23}=0,\ D_2=0 \tag{3.1}$$

rigid clamping, and the boundary coated with an electrode and grounded

$$u_3=0,\ E_1=0 \tag{3.2}$$

It can be shown that Green's functions in this case are given by (2.3) in which the coefficients b_k and d_k have the form

$$b_k = -\frac{1}{a\lambda_k}\left(e^{-\lambda_k|x_2-\xi_2|} - Ae^{-\lambda_k(x_2+\xi_2)}\right) \tag{3.3}$$
$$b_0 = \frac{1}{2ia\gamma_2}\left(e^{i\gamma_2|x_2-\xi_2|} - Ae^{i\gamma_2(x_2+\xi_2)}\right)$$
$$d_k = -\frac{1}{a\alpha_k}\left(e^{-\alpha_k|x_2-\xi_2|} + Ae^{-\alpha_k(x_2+\xi_2)}\right)\quad (k=1,2,\ldots)$$

Summing the corresponding series in (2.3) we obtain

$$G^*(x_1,x_2;\xi_1,\xi_2) = G(x_1-\xi_1,x_2-\xi_2) + A\left\{\frac{i}{2a\gamma_2}e^{i\gamma_2(x_2+\xi_2)} + \right.$$
$$+\frac{x_2+\xi_2}{2a} - \frac{1}{2\pi}\ln\left|4\sin\frac{\pi(\zeta+z)}{2a}\sin\frac{\pi(\zeta-\bar z)}{2a}\right| +$$
$$\left.+\frac{1}{a}\sum_{m=1}^{\infty} c_m^*(x_2+\xi_2)\cos\alpha_m\xi_1\cos\alpha_m x_1\right\}$$
$$E^*(x_1,x_2;\xi_1,\xi_2) = E(x_1-\xi_1,x_2-\xi_2) + \frac{A}{2\pi}\ln\left|\frac{\sin^1/_2\pi(\zeta-\bar z)/a}{\sin^1/_2\pi(\zeta+z)/a}\right| \tag{3.4}$$
$$c_m^*(x_2+\xi_2) = \frac{1}{\lambda_m}e^{-\lambda_m(x_2+\xi_2)} - \frac{1}{\alpha_m}e^{-\alpha_m(x_2+\xi_2)}\quad (m=1,2,\ldots)$$

Here the case $A = -1$ corresponds to a free half-layer bounded by vacuum and the case $A = 1$ corresponds to a clamped half-layer covered with a grounded electrode along the boundary $x_2 = 0$. For $A = 0$ we arrive at formulas (2.6) and (2.8) for a layer.

4. The displacement field U_3^* scattered by the cuts will be looked for in the form

$$U_3^*(x_1, x_2) = -2i \int_L p(\zeta) \left\{ \frac{\partial G^*}{\partial \zeta} d\zeta - \frac{\partial G^*}{\partial \bar{\zeta}} \partial \bar{\zeta} \right\}$$

$$p(\zeta) = \frac{[U_3^*]}{2}, \quad \zeta \in L \tag{4.1}$$

Here the unknown quantity $[U_3^*]$ has the meaning of the jump in the displacement amplitude U_3^* on L and $G^* = G^*(x_1, x_2; \xi_1, \xi_2)$ is defined in (3.4).

We will represent the function F as follows:

$$F(x_1, x_2) = \int_L f(\zeta) E^* ds \tag{4.2}$$

where $E^* = E^*(x_1, x_2; \xi_1, \xi_2)$ is given in (3.4) and ds is an element of the arc of the contour L.

To clarify the meaning of the density f in (4.2) we will first calculate the derivatives $\partial U_3^*/\partial z$, $\partial U_3^*/\partial \bar{z}$. As a result, after some reduction to improve the convergence of the corresponding series, we obtain

$$\frac{\partial U_3^*}{\partial z} = \frac{\pi}{8ia^2} \int_L p(\zeta) \operatorname{cosec}^2 \frac{\pi(\zeta - z)}{2a} d\zeta + \int_L p(\zeta)(R_1 e^{i\psi} + R_3 e^{-i\psi}) ds$$

$$\frac{\partial U_3^*}{\partial \bar{z}} = -\frac{\pi}{8ia^2} \int_L p(\zeta) \operatorname{cosec}^2 \frac{\pi(\bar{\zeta} - \bar{z})}{2a} d\bar{\zeta} + \int_L p(\zeta)(R_2 e^{i\psi} + R_4 e^{-i\psi}) ds, \tag{4.3}$$

$$R_1 = R_0 + \frac{A\pi}{8a^2} \operatorname{cosec}^2 \frac{\pi(\zeta + z)}{2a} - \frac{1}{2a}[A_1 - B_2 - i(A_2 + B_1)]$$

$$R_2 = -R_0 - Y - \frac{1}{2a}[A_1 + B_2 + i(A_2 - B_1)]$$

$$R_3 = -R_0 - \bar{Y} - \frac{1}{2a}[A_1 + B_2 - i(A_2 - B_1)]$$

$$R_4 = R_0 + \frac{A\pi}{8a^2} \operatorname{cosec}^2 \frac{\pi(\bar{\zeta} + \bar{z})}{2a} - \frac{1}{2a}[A_1 - B_2 + i(A_2 + B_1)]$$

$$R_0 = \frac{i\gamma_2}{4a}(e^{i\gamma_2|x_2 - \xi_2|} + Ae^{i\gamma_2(x_2 + \xi_2)}), \qquad R_m = R_m(\zeta, z)$$

$$Y = \frac{\pi}{8a^2}\left[\operatorname{cosec}^2 \frac{\pi(\zeta + \bar{z})}{2a} + A \operatorname{cosec}^2 \frac{\pi(\zeta - \bar{z})}{2a}\right]$$

$$A_1 = \frac{a\gamma_2^2}{4\pi} \ln\left|\frac{\sin^{1/2} \pi(\zeta + \bar{z})/a}{\sin^{1/2} \pi(\zeta - z)/a}\right| + \sum_{k=1}^{\infty}\left\{\frac{\alpha_k^2}{\lambda_k} e^{-\lambda_k|x_2 - \xi_2|} - \left(\alpha_k + \frac{\gamma_2^2}{2\alpha_k}\right) e^{-\alpha_k|x_2 - \xi_2|} - \right.$$

$$\left. - A\left[\frac{\alpha_k^2}{\lambda_k} e^{-\lambda_k(x_2 + \xi_2)} - \alpha_k e^{-\alpha_k(x_2 + \xi_2)}\right]\right\} \sin \alpha_k \xi_1 \sin \alpha_k x_1$$

$$A_2 = \sum_{k=1}^{\infty} \alpha_k [\beta^- \operatorname{sign}(x_2 - \xi_2) - A\beta^+] \sin \alpha_k \xi_1 \cos \alpha_k x_1$$

$$B_1 = \sum_{k=1}^{\infty} \alpha_k [\beta^- \operatorname{sign}(\xi_2 - x_2) - A\beta^+] \cos \alpha_k \xi_1 \sin \alpha_k x_1$$

$$B_2 = \frac{\gamma_2^2 |x_2 - \xi_2|}{4} - \frac{a\gamma_2^2}{4\pi} \ln\left|4 \sin \frac{\pi(\zeta - z)}{2a} \sin \frac{\pi(\zeta + \bar{z})}{2a}\right| +$$

$$+\sum_{k=1}^{\infty}\left\{\left(\alpha_k-\frac{\gamma_2^2}{2\alpha_k}\right)e^{-\alpha_k|x_2-\xi_2|}-\lambda_k e^{-\lambda_k|x_2-\xi_2|}+A\left[\alpha_k e^{-\alpha_k(x_2+\xi_2)}-\lambda_k e^{-\lambda_k(x_2+\xi_2)}\right]\right\}\times$$
$$\times\cos\alpha_k\xi_1\cos\alpha_k x_1$$
$$\beta^{\pm}=e^{-\lambda_k|x_2\pm\xi_2|}-e^{-\alpha_k|x_2\pm\xi_2|}$$

Calculating the limiting values of the functions (4.2) as $z\to\zeta_0\in L$ we obtain

$$\left[\frac{\partial F}{\partial\zeta}\right]=-\frac{e^{-i\psi}}{2}f(\zeta),\quad \left[\frac{\partial F}{\partial\bar\zeta}\right]=-\frac{e^{i\psi}}{2}f(\zeta) \tag{4.4}$$

Consequently, the last condition in (1.8) is satisfied automatically, while the penultimate condition, taking (4.3) and (4.4) into account, leads to the equations

$$f(\zeta)=\frac{2e_{15}}{\varepsilon_{11}{}^{S}}p'(\zeta),\quad p'(\zeta)=\frac{dp(\zeta)}{ds} \tag{4.5}$$

By substituting the limiting values of the function (4.3) and the derivatives $\partial F/\partial z$, $\partial F/\partial\bar z$ as $z\to\zeta_0\in L$ into the mechanical boundary condition (1.8) on one of the edges of L and making use of relation (4.5), we arrive at the following singular integro-differential equation:

$$\int_L p'(\zeta)g_1(\zeta,\zeta_0)\,ds+\int_L p(\zeta)g_2(\zeta,\zeta_0)\,ds=N(\zeta_0) \tag{4.6}$$
$$g_1(\zeta,\zeta_0)=\frac{1}{a}\operatorname{Im}\left[e^{i\psi_0}\left(\operatorname{ctg}\frac{\pi(\zeta-\zeta_0)}{2a}-\varkappa_0^2P\right)\right]$$
$$g_2(\zeta,\zeta_0)=2(1+\varkappa_0^2)\left[e^{i\psi_0}(e^{i\psi}R_1^0+e^{-i\psi}R_3^0)+\right.$$
$$\left.+e^{-i\psi_0}(e^{i\psi}R_2^0+e^{-i\psi}R_4^0)\right],\quad N(\zeta_0)=\frac{2X_3}{c_{44}{}^E}+2i\gamma_2\tau(1+\varkappa_0^2)\times$$
$$\times\sin\psi_0(e^{-i\gamma_2\xi_{20}}Ae^{i\gamma_2\xi_{20}}),\quad P=\operatorname{ctg}\frac{\pi(\bar\zeta+\zeta_0)}{2a}+A\left[\operatorname{ctg}\frac{\pi(\bar\zeta-\zeta_0)}{2a}+\right.$$
$$\left.+\operatorname{ctg}\frac{\pi(\zeta+\zeta_0)}{2a}\right],\quad \psi_0=\psi(\zeta_0),\ \zeta_0=\xi_{10}+i\xi_{20}\in L_j\quad (j=1,2,\ldots,k)$$

Here the kernel $g_1(\zeta,\zeta_0)$ is a singular (Hilbert type) kernel and $g_2(\zeta,\zeta_0)$, by virtue of the assumptions regarding L, can possess not more than a weak singularity; the functions $R_m^0=R_m(\zeta,\zeta_0)$ are defined in (4.3).

To fix the solution in the class of functions with derivatives that are not bounded on the ends of L [1], it is necessary to add the following additional conditions to (4.6):

$$\int_{L_j} p'(\zeta)\,ds=0\quad (j=1,2,\ldots,k) \tag{4.7}$$

5. Suppose that in the half-layer (the layer) there is one cut L, the parametric equation of which is $\zeta=\zeta(\delta)(-1\leqslant\delta\leqslant 1)$. We will represent the required density in the integro-differential equation (4.6) as follows:

$$p'(\zeta)=\frac{\Omega_0(\delta)}{s'(\delta)\sqrt{1-\delta^2}},\quad \Omega_0(\delta)\in H[-1,1],\ s'(\delta)=\frac{ds}{d\delta} \tag{5.1}$$

An asymptotic analysis of representations (4.3) and the derivatives $\partial F/\partial z$, $\partial F/\partial\bar z$ in the neighbourhood of the tip of the cut, taking (1.1) and (5.1) into account, enables us to obtain the stress intensity factor K_{III} [6] in the form (the upper sign relates to the start of the cut and the lower sign to the end of the cut)

$$K_{III}^{\mp} = \mp c_{44}^E \sqrt{\frac{\pi}{s'(\mp 1)}} \operatorname{Re}\{e^{-i\omega t}\Omega_0(\mp 1)\} \tag{5.2}$$

The asymptotic form of the normal component of the electric induction vector along the extension beyond the top of the cut is such that

$$D_n^{\mp} = D_1 \cos\psi(\mp 1) + D_2 \sin\psi(\mp 1) = \mp e_{15} \frac{\operatorname{Re}[e^{-i\omega t}\Omega_0(\mp 1)]}{\sqrt{2rs'(\mp 1)}} \tag{5.3}$$

where r is the distance to the tip.

The remaining electromagnetic quantities are bounded. In fact, we have from the equations of state (1.1)

$$\tau_n = c_{44}^E \frac{\partial u_3}{\partial n} - e_{15}E_n, \qquad D_n = e_{15}\frac{\partial u_3}{\partial n} + \varepsilon_{11}^S E_n \tag{5.4}$$

where D_n is the normal component of the electric induction on the arc L', as close to L as desired. Since $[\tau_n] = [D_n] = 0$ and the determinant of system (5.4) is non-zero, we obtain $[E_n] = 0$. Hence, the electric field E is continuously extendable through the cut and therefore is continuous everywhere.

6. The integro-differential equation (4.6), together with the additional condition (4.7), was reduced to a system of linear algebraic equations in the values of the functions $\Omega_0(\delta)$ at Chebyshev's interpolation nodes using the procedure described in [7] for the case when the half-layer (PZT-4 piezoelectric ceramics) contains a parabolic cut $\xi_1 = p_0 + p_1\delta$, $\xi_2 = h + p_2\delta^2$, $\delta \in [-1, 1]$. The approximate values of the function Ω_0 were calculated for a number of nodes $n = 9$, 11 and 13, $m = 9$ terms were retained in this series. Any further increase in the parameters n and m hardly affected the accuracy of the results.

Suppose $X_3 = 0$ (the edges of the cuts are free from forces), while an *SH* displacement wave is incident from infinity onto a rectilinear cut. The change in value of $\alpha^+ = c_{44}^E|\Omega_0(1)|/T_{23}^0\sqrt{ls'(1)}$ as a function of the normalized wave number $\gamma_2{}^*l = \gamma_2 l\sqrt{1+\varkappa_0^2}$, $a = 1$ m ($2l$ is the length of the cut) for $h/a = p_0/a = 0.5$ and $p_1/a = 0.2$, is shown in Fig. 2. Curves 1 and 2 were drawn for values of the parameter $A = 1$ and -1, respectively, and the continuous curves relate to the case of ceramics, while the dashed curves relate to the case where $\varkappa = 0$ (an isotropic material). Here T_{23}^0 is the modulus of the amplitude of the stress τ_{23} in the incident wave.

Knowing the quantities of $\alpha^{\mp}$ and $\delta^{\mp} = \arg[\Omega(^{\mp}1)]$ we can determine the stress intensity factor K_{III} from the formula

$$K_{III}^{\mp} = \mp T_{23}^0 \sqrt{\pi l}\alpha^{\mp} \cos(\omega t - \delta^{\mp})$$

In Fig. 3 we show graphs of the quantity $\alpha^+ = c_{44}^E|\Omega_o(1)|/|X_3|\sqrt{ls'(1)}$ as a function of the

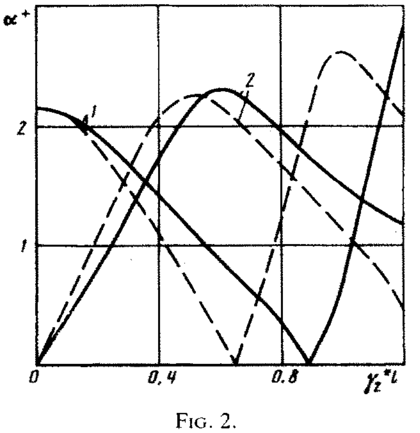

FIG. 2.

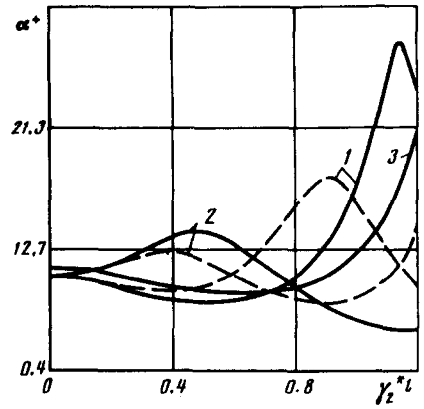

FIG. 3.

parameter $\gamma_2{}^*l$ for the case when there is no radiation from infinity, and a shear load ($\tau = 0$, X_3 = const), which varies harmonically with time, acts on the edges of the cut. Curves 1 and 2 are drawn for the same parameters and the same correspondence as in Fig. 2; curve 3 relates to the value of the parameter $p_2/a = -0.1$ (a curvilinear cut).

The stress intensity factor was calculated in this case from the formula

$$K_{III}{}^{\mp} = \mp |X_3| \sqrt{\pi l} \alpha^{\mp} \cos(\omega t - \delta^{\mp})$$

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